# 1 Period optimisation, models and cost functions

State : $\begin{bmatrix} x_k^T & \Theta_k^T & \dot{x}_k^T & \dot{\Theta}_k^T & P_{feet} & dt \end{bmatrix}^T$ , x12 x8 x1 ($P_{feet} = [x_0, y_0, \dots x_3, y_3]$)

Period : $\Delta T$ = Nb nodes x dt

## 1.1 Augmented model

In the same optimal problem (with the same dynamic and cost function), the state vector is augmented to optimise the position of the feet : $Y = \begin{bmatrix} X \\ P \\ dt \end{bmatrix}$.

The P vector contains the position of the feet in X,Y axis (vector of size 8) : $P^T = \begin{bmatrix} x_0 & y_0 & \dots & x_3 & y_3 \end{bmatrix}^T$
The size of the augmented vector is now 21. Here is the new dynamic :

$$Y_{k+1} = \begin{bmatrix} X_{k+1} \\ P_{k+1} \\ dt_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0_9 \\ 0_{13} & I_9 \end{bmatrix} \begin{bmatrix} X_k \\ P_k \\ dt \end{bmatrix} + \begin{bmatrix} B \\ 0_{9\times 12} \end{bmatrix} U_k$$

Here $X = X_k$ to not overload the writing.

$$\begin{aligned} Cost_k =& \frac{1}{2}\delta_1 ||X - X_{ref}||^2 + \frac{1}{2}\delta_2 ||U||^2 \\ &+ \frac{1}{2}\delta_3 \sum_{i=1}^{4} ||(f_{x,i} - \mu f_{z,i})^+||^2 + ||(f_{y,i} - \mu f_{z,i})^+||^2 + ||(-f_{z,k} + f_{min})^+||^2 + ||(f_{z,k} - f_{max})^+||^2 \end{aligned}$$

- $\delta_1$ State Cost
- $\delta_2$ Cost on the norm of the forces
- $\delta_3$ Friction cost

## 1.2 Feet Step model

Here is the model inserted between dynamic model. As only two foot need to be placed at a time, the input command is size 4 : $U_k^T = \begin{bmatrix} \Delta x_1 \ \Delta y_1 \ \Delta x_2 \ \Delta y_2 \end{bmatrix}$.

$$Y_{k+1} = \begin{bmatrix} X_{k+1} \\ P_{k+1} \\ dt \end{bmatrix} = \begin{bmatrix} I_{12} & 0_8 & 0 \\ 0_{12} & I_8 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} X_k \\ P_k \\ dt \end{bmatrix} + \begin{bmatrix} 0_{12\times 4} \\ B_2 \\ 0 \end{bmatrix} U_k$$

In this model, the initial state vector $X_k$ does not change and the foot position is adjusted : $P_{k+1} = P_k + \Delta P$. The state dt does not change.

Approximate foot trajectory in X,Y plan with a polynomial function of degree 5

Initial conditions : $x(0) = x_0$ , $x(\Delta T) = x_1$ , $\dot{x}(x_0) = 0$ , $\ddot{x}(x_0) = 0$ ...

$$x(t) = x_0 + \frac{10}{\Delta T^3}(x_1 - x_0)t^3 - \frac{15}{\Delta T^4}(x_1 - x_0)t^4 + \frac{6}{\Delta T^5}(x_1 - x_0)t^5$$

The maximum speed during flying phase :

$$\dot{x}_{max} = \frac{15}{8\Delta T}(x_1 - x_0)$$

To keep $V^2 = V_x^2 + V_y^2 < V_{lim}^2$

$$cost_{\Delta U} = \frac{1}{2}\delta(\Delta p_x^2 + \Delta p_y^2 - \beta dt^2)^+$$

$$Cost_k = +\frac{1}{2}\delta_4[(\Delta u_1^2 + \Delta u_2^2 - \beta x_{21}^2)^+ + (\Delta u_3^2 + \Delta u_4^2 - \beta x_{21}^2)^+] + \frac{1}{2}\delta_5||U||^2$$

— $\delta_4$ Speed Cost
— $\delta_5$ Cost on the norm of the delta feet ( $P_{k+1} = P_k + \Delta P$)

### 1.2.1 Dt change model

Nothing is changed, dt = U.

$$Y_{k+1} = \begin{bmatrix} X_{k+1} \\ P_{k+1} \\ dt \end{bmatrix} = \begin{bmatrix} I_{12} & 0_8 & 0 \\ 0_{12} & I_8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_k \\ P_k \\ dt \end{bmatrix} + \begin{bmatrix} 0_{12} \\ 0_8 \\ 1 \end{bmatrix} U_k$$

$$Cost_k = \frac{1}{2}\delta_6(||(dt_{min} - U)^+||^2 + ||(U - dt_{max})^+||^2)$$

— $\delta_6$ Command upper/lower bound, here dt = U = $x_{k+1,21}$

### 1.2.2 Insertion of dt change and step model wrt the gait matrix

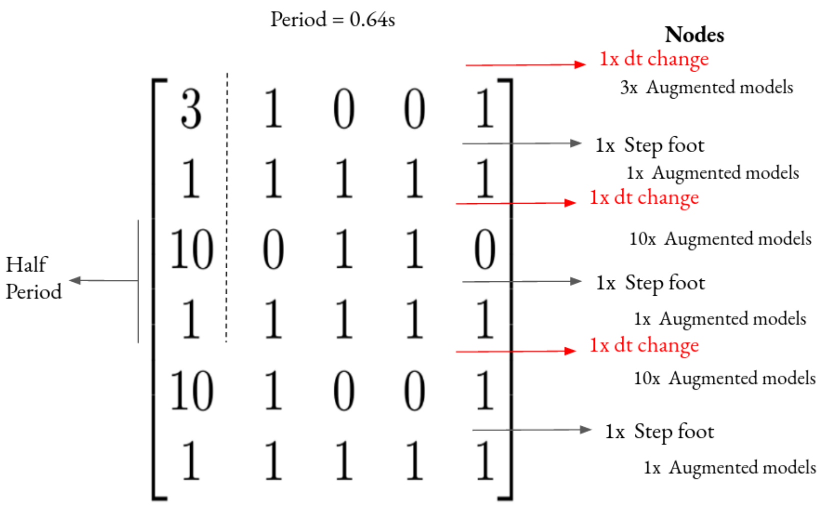


FIGURE 1 – "Explanation of the test, dt = 0.02s"