

1 Cost function : term over the shoulder height

A new term is added to the cost function to take into account the height of each shoulder and avoid that one of the feet no longer touch the ground. This usually happens with the non linear model and lateral velocity targets. The height increases and it is coupled with a high roll angle.

The angular position vector is $\Theta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} roll \\ pitch \\ yaw \end{bmatrix}$. The passage between the local frame and the global frame is described using Z-Y-X Euler angles and the following rotation : $R = R_z(\psi)R_y(\theta)R_x(\phi)$. The shoulder belongs to X,Y plan of the local frame ($p_z = 0$). Thus, we have the relation between local and global frame (without the translation z, height of the robot) :

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 0 \end{bmatrix}$$

Thus, we have the height of the shoulder in global frame from the shoulder position in the local frame and the rotation angles (with the small angles approximation, the maximum roll obtained is around 8 degrees, the approximation is still valid) :

$$\begin{cases} p'_x = p_x \cos(\theta) \cos(\psi) + p_y [\sin(\theta) \cos(\psi) \sin(\phi) - \sin(\psi) \cos(\phi)] \\ p'_y = p_x \cos(\theta) \sin(\psi) + p_y [\sin(\theta) \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi)] \\ p'_z = -p_x \sin(\theta) + p_y \cos(\theta) \sin(\phi) \end{cases}$$

Position of the shoulder in optimisation frame (+ translation) :

$$\begin{cases} x_{sh} = x + p_x - p_y \psi \\ y_{sh} = y + p_y + p_x \psi \\ z_{sh} = z + p_y \phi - p_x \theta \end{cases} \quad (1)$$

Then, the cost function is :

$$Cost = \frac{1}{2} \delta \| ((x_{sh} - x_c)^2 + (y_{sh} - y_c)^2 + z_{sh}^2 - d_{lim}^2)^+ \|^2$$

where $\|y^+\|^2 = \|y\|^2$ if $y > 0$ and 0 otherwise.