1 Foot step planner

1.1 Optimal problem, initial formulation

$$X_{k+1} = AX_k + BU_k$$

The state vector is : $X_k = \begin{bmatrix} x_k^T & \Theta_k^T & \dot{x}_k^T & \dot{\Theta}_k^T \end{bmatrix}^T$. x_k is the position vector of the CoM (x,y,z), Θ_k is the rotation angles vector (roll, pitch, yaw), $\dot{x}_k$ the linear velocity vector and $\dot{\Theta}_k$ the angular velocity (X_k of size 12).
The command vector is $U_k^T = \begin{bmatrix} f_{x1} \; f_{y1} \; f_{z1} \dots \; f_{xn} \; f_{yn} \; f_{zn} \end{bmatrix} = \begin{bmatrix} f_1^T & f_2^T & f_3^T & f_4^T \end{bmatrix}$ where f_{x1} is the ground reaction force among the x-axis in the local frame of the first foot.
There are three terms in the cost function : a quadratic term on the norm of the forces $\frac{1}{2}||U||^2$, a quadratic term between the state and the desired state : $\frac{1}{2}||X - X_{ref}||^2$ and a third quadratic term to make the forces respect the friction cone.

1.2 Optimal problem, formulation of the foot step planner

1.2.1 Augmented model

In the same optimal problem (with the same dynamic and cost function), the state vector is augmented to optimise the position of the feet : $Y = \begin{bmatrix} X \\ P \end{bmatrix}$.
The P vector contains the position of the feet in X,Y axis (vector of size 8) : $P^T = \begin{bmatrix} x_1 & y_1 & \dots & x_4 & y_4 \end{bmatrix}^T$
The size of the augmented vector is now 20. Here is the new dynamic :

$$Y_{k+1} = \begin{bmatrix} X_{k+1} \\ P_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0_8 \\ 0_{12} & I_8 \end{bmatrix} \begin{bmatrix} X_k \\ P_k \end{bmatrix} + \begin{bmatrix} B \\ 0_{8\times 12} \end{bmatrix} U_k$$

In this model, the position of the feet does not change : $P_{k+1} = P_k$ because the feet are on the ground. The cost function is composed of :

- Quadratic cost on state parameters : $\frac{1}{2}||X_k - X_{ref}||^2$
- Quadratic cost on force input : $\frac{1}{2}||F_k||^2$
- Quadratic cost for the friction cone.
- Quadratic cost to keep the position of foot below the shoulder : $\frac{1}{2}||P_k - P_{shoulder}||^2$
- Quadratic cost to stop the optimisation of foot placement by using the position of foot computed at the previous control cycle : $\frac{1}{2}||P_k - P_{last,pos}||^2$

The last term is used to stop optimising when the foot is close to reaching the ground. It avoids destabilisation at the end of the movement. This term is activated only on one node, after the step one.

1.2.2 Step model

Another model is inserted when the position of the foot must change. For each control cycle, here is the gait matrix that indicates which foot will be in contact with the ground for each node. The first position of feet is considered known (predicted by the previous control cycle). When a foot need to be placed on the ground (red cycle on the figure), a different model of dynamic is inserted between the dynamic model node.
First column is the number of node spent on that position. The corresponding line shows which feet are in contact with the ground (2nd line : all feet in contact with the ground).

$$
S = \begin{array}{c@{}l}
\left[\begin{array}{c:cccc}
1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 \\
7 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
6 & 1 & 0 & 0 & 1
\end{array}\right]
&
\begin{array}{l}
\rightarrow \text{Step} \\
\\
\rightarrow \text{Step} \\
\\
\\
\end{array}
\end{array}
$$

FIGURE 1 – "Gait Matrix, position to optimise"

Here is the model inserted between dynamic model. As only two foot need to be placed a a time, the input command is size 4 : $U_k^T = \begin{bmatrix} \Delta x_1 & \Delta y_1 & \Delta x_2 & \Delta y_2 \end{bmatrix}$.

$$Y_{k+1} = \begin{bmatrix} X_{k+1} \\ P_{k+1} \end{bmatrix} = \begin{bmatrix} I_{12} & 0_8 \\ 0_{12} & I_8 \end{bmatrix} \begin{bmatrix} X_k \\ P_k \end{bmatrix} + \begin{bmatrix} 0_{12\times 4} \\ B_2 \end{bmatrix} U_k$$

In this model, the initial state vector X_k does not change and the foot position is adjusted : $P_{k+1} = P_k + \Delta P$. The cost function is composed of :

— Quadratic cost on state parameters : $\frac{1}{2}||X_k - X_{ref}||^2$

— Quadratic cost to minimise the jump between foot positions : $\frac{1}{2}||\Delta P||^2$

— Quadratic cost to keep the position of foot below the shoulder : $\frac{1}{2}||P_k - P_{shoulder}||^2$